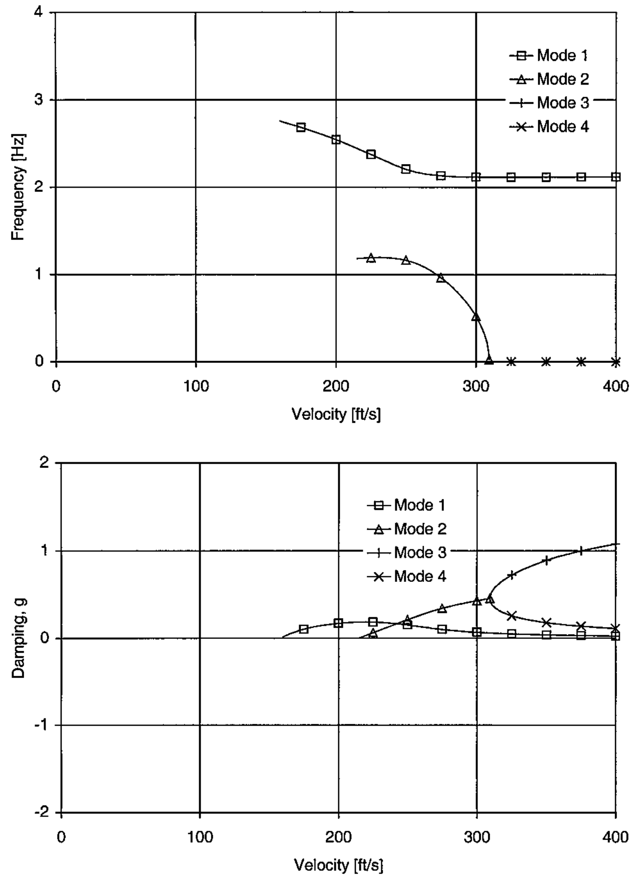

Fig. 4 Unstable roots of the exact equation of motion.

(mode 1), the elastic plunge mode (mode 2), and the rigid plunge mode (mode 3). The pitch mode flutters at 158.4 ft/s, and the elastic plunge mode flutters at 215.0 ft/s. The frequency of the elastic plunge mode goes to zero between 371 and 372 ft/s with a positive divergence rate $g = 0.26$. The mode then splits into two real roots, one becoming more unstable and the other becoming less unstable.

The unstable roots of Eq. (1) are shown in Fig. 4. The oscillatory roots were found by a mode tracking procedure starting at the flutter points. The real roots were found by a root search procedure with the aerodynamic matrix calculated for exponentially diverging motion. The solution is qualitatively similar to the solution of Rodden, Harder, and Bellinger's form of the flutter equation[4]; however, the frequency of the plunge mode goes to zero between 309.3 and 309.4 ft/s and the initial divergence rate $g = 0.46$.

It was verified that the divergence roots of Hassig's form[3] with order six (Fig. 1) and five (Fig. 2) migrate smoothly to the exact result (Fig. 4) by substituting $Q(fpc/2U)$ for $Q(pc/2U)$ in Eq. (1) and letting f vary from 0 to 1. The only surprising result was that the complete parabolic structure in Fig. 1 does not migrate, but first divides along the velocity axis. It was also verified that the solution of Eq. (1) with order six was identical to the solution with order five.

Discussion

In the solution of Hassig's form[3] of the flutter equation with order six the divergence roots appear independently of the other roots of the system. In the solution of Hassig's form with order five, the divergence roots are linked to the rigid plunge mode by a bifurcated root. In both these cases the instabilities at 158.4 and 215.0 ft/s would be regarded as flutter, whereas the origin of the real roots would be regarded as the onset of divergence.

In the solutions of Rodden, Harder, and Bellinger's form of the flutter equation[4] as well as the exact equation of motion, the real roots originate where the frequency of the elastic plunge mode goes to zero. It is open to interpretation whether the onset of divergence is where the root becomes unstable or where it becomes nonoscillatory. It is, however, not valid to compare the divergence speed predicted by the quasi-static unrestrained divergence analysis to the flutter point.

Summary

In Ref. 1 the accuracy of the quasi-steady unrestrained divergence analysis was questioned. Hassig's form[3] of the flutter equation with order six, which ignores the effect of exponentially decaying or growing motion on the aerodynamics, agrees exactly with the quasi-static unrestrained divergence analysis. This suggests that the flaw in quasi-static unrestrained divergence analysis is the assumption of quasi-static aerodynamics.

References

[1]Rodden, W. P., and Bellinger, E. D., "Unrestrained Aeroelastic Divergence in a Dynamic Stability Analysis," *Journal of Aircraft*, Vol. 19, No. 9, 1982, pp. 796–797.

[2]Rodden, W. P., "Aeroelastic Divergence of Unrestrained Vehicles," *Journal of Aircraft*, Vol. 18, No. 12, 1981, pp. 1072–1073.

[3]Hassig, H. J., "An Approximate True Damping Solution of the Flutter Equation by Determinant Iteration," *Journal of Aircraft*, Vol. 8, No. 11, 1971, pp. 885–889.

[4]Rodden, W. P., Harder, R. L., and Bellinger, E. D., "Aeroelastic Addition to NASTRAN," NASA CR 3094, 1979.

[5]Jones, W. P., "Aerodynamic Forces on Wings in Non-Uniform Motion," Repts. and Memoranda 2117, 1945, Aeronautical Research Council, London.

[6]Van Zyl, L. H., "Aeroelastic Divergence and Aerodynamic Lag Roots," *Journal of Aircraft*, to be published.

[7]Rodden, W. P., and Johnson, E. H., "MSC/NASTRAN Aeroelastic Analysis User's Guide," MacNeal-Schwendler Corp., Los Angeles, CA, 1994.

Simplified Treatment of Unsteady Aerodynamics for Lifting Rotors

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Introduction

IT is sometimes useful in computing blade airloads to have approximate means to account for the nonstationary flow effects resulting from the complex time varying flowfields in which lifting rotors operate. This Note suggests such an approximation and compares the results with solutions obtained by more rigorous methods for the following two cases: 1) a two-dimensional oscillating airfoil with a shed wake remaining in the plane of the airfoil and 2) a two-dimensional oscillating airfoil including a returning shed wake located below the airfoil.

Reference 1 has formed the basis of most nonstationary flow aerodynamic analyses in which the wake may be assumed to remain in the plane of the airfoil. Reference 2 demonstrated the importance of considering the returning wake for rotors in vertical flight. It was shown that the blade damping could be reduced to very low values at integers of the ratio of blade frequency to the forcing frequency and could approach zero under conditions of no inflow. Both of these treatments are frequency based. It has been recognized for some time that, for rotors, a direct time-dependent treatment could be advantageous, as discussed in Ref. 3. This is particularly so in view of the highly variable changes in load, both temporal and spatial, associated with such a phenomenon as blade and vortex interaction, a primary contributor to the higher harmonic blade loadings of cruising flight.

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A simplified method of treating the unsteady aerodynamic effects, due to both the shed and trailed wakes, in the time domain, was introduced in Ref. 4. It is the purpose of this Note to compare results obtained with this method, in the frequency domain, with results obtained by the methods of Refs. 1 and 2. The portion of the wake that may be assumed to remain in the plane of the airfoil will be referred to as the near wake. The returning wake below the rotor will be referred to as the far wake.

Two-Dimensional Solutions for Near Wake

In classical two-dimensional non stationary flow theory, the circulatory airloads are modified by a frequency-dependent function, $C(k) = F + iG$. F is the ratio of the loading to the quasi-static loading and $\tan^{-1}(G/F)$ the phase shift of the loading relative to the forcing function. The reduced frequency is defined as $k = \omega b/v$, where ω is the frequency, b the half-chord, and v the blade velocity at the radius r. In the case of hovering flight, $v = \Omega r$, where Ω is the blade rotational speed. For harmonic loading, $\omega = n\Omega$, where n is the harmonic of interest, whence $k = nb/r$.

In the simplified treatment the rotor blade may be represented by a lifting line in view of the usually high aspect ratios of rotor blades. The induced flow is computed by iteration on summations of the form

$$u = \frac{1}{2\pi}\sum_{k=1}^{m}\sum_{j=1}^{k}\frac{\Delta\Gamma}{\Delta s}\cdot\log\left[\frac{\Delta s\cdot(k-j+1)+e}{\Delta s\cdot(k-j)+e}\right] \qquad (1)$$

where u is the induced flow and Γ is the bound circulation. $\Gamma = 2\pi bv(\theta - u/v)$, where θ is the forcing function equal to $\Theta\sin(n\cdot\Delta s\cdot j)$ an assumed harmonic pitch change. The term e is needed to avoid the singularity when $j = k$ and was shown in Ref. 5 to be a function of k. In the simplified treatment, e will be assumed constant. The logarithmic form of the shed vortex contribution to u, obtained by integration over Δs, is used to minimize sensitivity of the solution to the degree of discretization, as determined by m, the number of stations in the summations. The summations may be conducted over a half revolution to multirevolutions without appreciably changing the results. The choice of Δs will depend on the time interval required to capture an expected abrupt change in loading. However, to determine $C(k)$ from the solution in the time domain, for a comparison with the frequency-domain solution of Ref. 1, a small Δs is required, particularly at the higher harmonics. Figure 1 shows this comparison for reduced frequencies corresponding to harmonics up to $n = 20$ for a rotor with $b/r = 0.025$, $\Delta s = 2\pi/m/n$, $m = 100$, and $e = b/2$, the rear neutral point of a full chord blade, and for $e = b$, the blade midpoint. Best results for the lift deficiency function F is with $e = b/2$ and for the phase shift, determined by G, with $e = b$. Exact definition of the phase, particularly at the higher frequencies, may be important in dynamic stability analyses, otherwise a value of e of $b/2$ would appear to be a reasonable compromise for the computation of blade loadings.

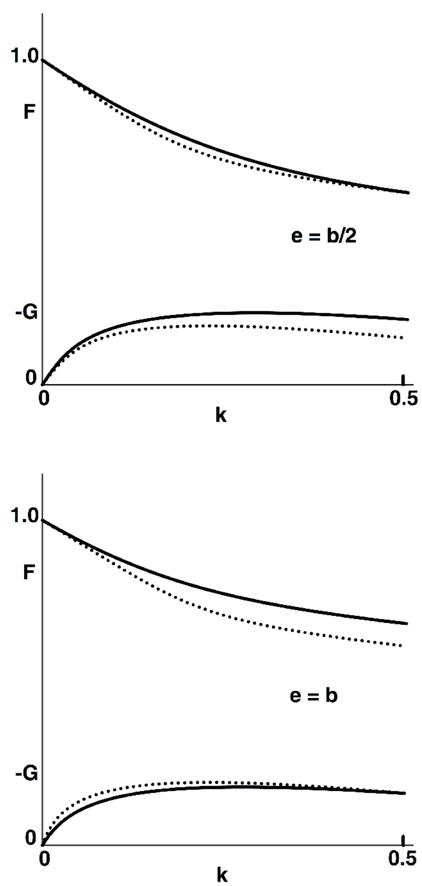

Fig. 1 Comparison of Ref. 1 theory with results from Eq. (1): . . . . ., Ref. 1, and ——, Eq. (1).

Two-Dimensional Solution for Far Wake

It is of interest to examine the application of the simplified treatment to the model, including the far wake, of Ref. 2. Reference 6 introduced a similar model, but one in which the blade was treated as a lifting line when computing the contributions of the far wake. $C(k)$ then became the same as in Ref. 1, in terms of the Bessel functions J_0, J_1, Y_0, and Y_1, with one additional term, rt, to take into account the far wake extending to infinity,

$$C(k) = (J_1 - Y_1)/[(J_1 - Y_0) + (J_0 - Y_1 + rt)i] \qquad (2)$$

where $rt = 2/(e^{hk} - 1)$ and h is the spacing between far wake elements below the rotor divided by the semichord b. It was shown in Ref. 6, Fig. 4 (in Fig. 4 of Ref. 6 the scale of $-G$ should be 0.25, not 0.5) that this gives an excellent approximation for F but, as could be expected, fails to predict G as the wake approaches the blade, as shown in Fig. 2 here for the case of $h = 1$. However, with the proposed simplified solution, the agreement is good for both values of h. In this solution the contribution uf of the far wake to the induced flow is determined from summations of the form

$$uf = \frac{1}{2\pi}\sum_{k=1}^{m}\sum_{j=1}^{2m}\sum_{p=1}^{w}\frac{\Delta\Gamma f}{\Delta s}\frac{sf}{sf^2 + d^2}\Delta s \qquad (3)$$

where $sf = \Delta s(k - j + 0.5) + e$ and d is p times h, the distance of a far wake element below the rotor. The circulation in the far wake varies as $\Gamma f\cdot\sin(n\cdot\Delta s\cdot j + \phi)$ where Γf is the maximum value of Γ in the near wake as determined from Eq. (1) and ϕ is the phase shift. Hence, $\Delta\Gamma f/\Delta s = \Gamma f\cdot n\cdot\cos(n\cdot\Delta s\cdot j + \phi)$. A value of $w = 6$ is in general sufficient. As in the case of Eq. (1), the numerical results are

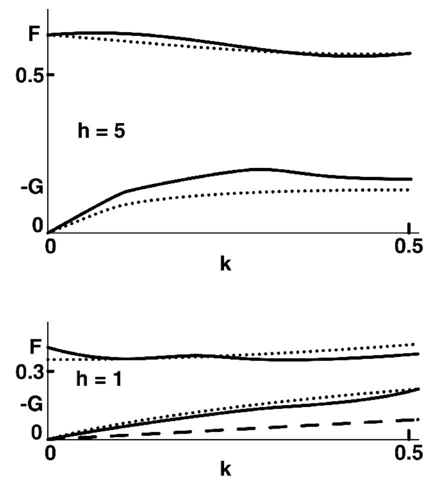

Fig. 2 Comparison of Ref. 2 theory with results from Eqs. (2) and (3): . . . . ., Ref. 2; - - - -, Eq. (2); and ——, Eq. (3).

not particularly sensitive to the length of shed wake included in the summations, particularly in the case of the far wake, and m may be truncated as required by the need to reduce computational intensity. The insensitivity of blade loading to the amount and curvature of the wake led to the development of the fast free wake concept, reviewed in Ref. 4. In the limit, the solutions approach the closed-form three-dimensional solutions, including both shed and trailing wakes, given in Ref. 6 for a hovering rotor with an infinite number of blades and an ideal twist and, hence, constant circulation along the blade, of $F = 1/(1 + \pi/h)$ and $G = 0$. This gives the same results as the actuator disk momentum theory developed by Rankine and Froude.

Summary

In summary, the proposed approach consists, in a time-marching computation, of stopping the integrations over the shed wakes for the induced flows at one-quarter of the blade chord $b/2$ from the lifting line blade representation or, in the case of a full blade representation, from the three-quarter chord point on the blade, the rear neutral point. This is not surprising in view of the importance of the induced flow, as computed at this point, in determining the circulatory airloads in both unsteady and steady flows.[7] The simplified treatment appears to agree well with results obtained from the more precise treatments. It has become evident that more complete models, including the many complex problems of rotor aerodynamics, such as aeroelastic and real fluids effects, are needed in engineering design and could well require a prohibitively high degree of computational intensity unless reliable simplifications are introduced where possible. Such simplifications also help in clarifying the physics of the problem and in defining the sensitive parameters, thus providing timely guidance in the solution of problems experienced, for example, during flight testing.

References

[1]Theodorsen, T., "General Theory of Aerodynamic Instability and the Mechanism of Flutter," NACA Rept. 496, 1935.

[2]Loewy, R. G., "A Two Dimensional Approach to the Unsteady Aerodynamics of Rotary Wings," *Journal of the Aeronautical Sciences*, Vol. 24, No. 2, 1957, pp. 82–98.

[3]Friedmann, P., "Arbitrary Motion Unsteady Aerodynamics and Its Application to Rotary-Wing aeroelasticity," *Journal of Fluids and Structures*, Vol. 1, No. 1, 1987, pp. 71–93.

[4]Miller, R. H., "The Development and Application of the Fast Free Wake Technique," *Proceedings of the Sixth International Workshop on Dynamics and Aeroelastic Stability Modeling of Rotorcraft*, US Army Research Office, Univ. of California, Los Angeles, Nov. 1995.

[5]Miller, R. H., "Unsteady Airloads on Helicopter Rotor Blades," Fourth Cierva Memorial Lecture, *Journal of the Royal Aeronautical Society*, Vol. 66, No. 640, 1964, pp. 217–229.

[6]Miller, R. H., "Rotor Blade Harmonic Air Loading," *AIAA Journal*, Vol. 2, No. 7, 1964, pp. 1254–1268.

[7]Bisplinghoff, R. L., Ashley, H., and Halfman, R. L., *Aeroelasticity*, Addison Wesley Longman, Reading, MA, 1957, p. 283.